Math 4200
Wednesday November 11

4.1-4.2 homework discussion; exam review

Announcements:

The floor is open for discussion of the homework due Friday! Groups? In the second portion of class we'll go over the review outline in Monday's notes.

There is a section 4.3 homework assignment due next week Friday -see page 2. It's applications of the Residue Theorem to computing definite integrals, many of which are not accessible using regular Calculus techniques.

Math 4200-001
Homework 12
4.3
Due Friday November 20 at 11:59 p.m.

4.3: 1, 2, 4, 6, 10, 14, 17, 20ab.
There are a lot of good worked examples in the text. In problem 6 you may use entry #5 on the Definite integral table 4.3.1, page 296. The text explains why this table entry is true on pages 289-293 and summarizes it as Proposition 4.3.16. We'll also discuss 6, 14 some in class on Monday

Math 4200-001
Homework 11
4.1-4.2
Due Friday November 11 at 11:59 p.m.
Exam will cover thru 4.2

4.1 1de, 3, 5, 7ab, 9
4.2 2 (Section 2.3 Cauchy's Theorem), 3, 4, 6, 9, 13.

w11.1 (extra credit) Prove Prop 4.1.7, the determinant computation for the residue at an order k pole for $f(z) = \dfrac{g(z)}{h(z)}$ at z_0, where $g(z_0) \neq 0$. (Hint: it's Cramer's rule for a system of equations.)

4.1.1 Find residues:

d) $\dfrac{1 + e^z}{z^4}$, $z_0 = 0$; e) $\dfrac{e^z}{(z^2 - 1)^2}$, $z_0 = 1$

4.1.3 Show by example that $Res(f(z)^2, z_0) \neq (Res(f(z), z_0))^2$ in general.

4.1.5 What fails in this reasoning: Let

$$f(z) = \frac{1 + e^z}{z^2} + \frac{1}{z}$$

Since $f(z)$ has a pole at $z = 0$ the residue of f at that point is the coefficient of $\frac{1}{z}$ there, namely 1.

4.1.7 Find all singular points and residues:

a) $\frac{1}{z^3(z+4)}$ b) $\frac{1}{z^2 + z + 1}$ c) (not assigned) $\frac{1}{z^3 - 3}$

4.1.9 Find the residue of $\frac{1}{z^2 \sin(z)}$ at $z = 0$

4.2 2 (Section 2.3 Cauchy's Theorem), 3, 4, 6, 9, 13.

2) Deduce Cauchy Integral Formula from Residue Theorem

3) Evaluate $\int_\gamma \frac{z}{z^2 + 2z + 5} dz$ where γ is the unit circle.

4) Find $\int_\gamma \frac{1}{e^z - 1} dz$ where γ is the circle of radius 9 and center zero.

6) Show $\int_\gamma \frac{5z - 2}{z(z-1)} dz = 10 \pi i$ where γ is any circle of center 0 and radius greater than 1.

9) Evaluate

a) $$\int_{|z| = \frac{1}{2}} \frac{dz}{z(1-z)^3}$$

b) $$\int_{|z| = \frac{1}{2}} \frac{e^z\, dz}{z(1-z)^3}$$

13a) Find $Res\left(\frac{(z-1)^3}{z(z+2)^3}; \infty\right)$ Recall, $Res(f;\ \infty) := Res\left(-\frac{1}{z^2} f\left(\frac{1}{z}\right); 0\right)$

13b) Compute $\displaystyle\int_{|z| = 3} \frac{(z-1)^3}{z(z+2)^3}\, dz$ two ways.